Package ‘RTDE’

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Type Package

Title Robust Tail Dependence Estimation

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Description Robust tail dependence estimation for bivariate models. This package is based on two papers by the authors:'Robust and bias-corrected estimation of the coefficient of tail dependence' and 'Robust and bias-corrected estimation of probabilities of extreme failure sets'. This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

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dataRTDE *Data object used for a Tail Dependence model*

Description

Data object used for a Tail Dependence model.

Usage

```
dataRTDE(obs, simu.nb, simu.marg=c("ufrechet", "upareto"),
    simu.cop=c("indep", "FGM", "Frank"), simu.cop.par=NULL,
    contamin.eps=NULL, contamin.method=c("NA","max+","+"),
    contamin.marg=c("ufrechet", "upareto"),
    contamin.cop=c("indep", "FGM", "Frank"),
    contamin.cop.par=NULL, control=list())


## S3 method for class 'dataRTDE'
print(x, ...)
## S3 method for class 'dataRTDE'
summary(object, ...)
## S3 method for class 'dataRTDE'
plot(x, which=1:2, ...)
```

Arguments

`obs`	bivariate numeric dataset.
`simu.nb`	a numeric for the sample size of simulated data.
`simu.marg`	a character string for the marginal distribution: either `"ufrechet"` (default) or `"upareto"`.
`simu.cop`	a character string ofr the copula: either `"indep"` (default), `"FGM"` or `"Frank"`.
`simu.cop.par`	a numeric for the copula parameter, default to `NULL`.
`contamin.eps`	a numeric for the percentage (of `simu.nb`) of contaminated data.
`contamin.method`	a character string for the contamination method: either `"NA"` (default), `"max+"` or `"+"`.
`contamin.marg`	a character string for the marginal distribution: either `"ufrechet"` (default) or `"upareto"`.
`contamin.cop`	a character string ofr the copula: either `"indep"` (default), `"FGM"` or `"Frank"`.
`contamin.cop.par`	a numeric for the copula parameter, default to `NULL`.
`control`	A list of control paremeters. Unused.
`x, object`	an R object inheriting from `"dataRTDE"`.

...	arguments to be passed to subsequent methods.
which	an integer (1 or 2) to specify whether to plot in original scale or unit-Pareto scale, respectively.

Details

The function dataRTDE handles empirical or simulated data and may add a contamination.

Empirical data When obs is provided, dataRTDE just wraps the two-column matrix $(X_i, Y_i)_i$.

Simulated data When simu.XXX are provided, dataRTDE simulates random vectors $(X_i, Y_i)_i$ from the copula simu.cop with parameter simu.cop.par and marginal simu.marg.

Note that end-user must choose between empirical data (obs is provided) and simulated data (simu.XXX are provided). Not both can be provided. In addition to data handling $(X_i, Y_i)_i$, a contamination can be processed by adding new simulated points $(\tilde{X}_i, \tilde{Y}_i)_i$ when contamin.method != "NA". Those points $(\tilde{X}_i, \tilde{Y}_i)_i$ are simulated from the copula contamin.cop with parameter contamin.cop.par and marginal contamin.cop.par. If contamin.method != "+", the points $(\tilde{X}_i, \tilde{Y}_i)_i$ are the contaminations, while if contamin.method != "max+" the contaminations are obtained by adding the component-wise maximum of the data: $(\tilde{X}_i + X_{n,n}, \tilde{Y}_i)_i + Y_{n,n}$, where $X_{n,n} = max(X_1, ..., X_n)$, idem for $Y_{n,n}$.

Value

dataRTDE returns an object of class "dataRTDE" having the following components:

n rownumber of data.

n0 rownumber of contamin.

data original or simulated data.

contamin contaminated data.

Author(s)

Christophe Dutang

References

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Volume 57, Insurance: Mathematics and Economics

This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

See Also

See fitRTDE for the fitting process and zvalueRTDE for the z-value computation.

Examples

```
#####
# (1) simulation

n <- 100
x <- dataRTDE(simu.nb=n, simu.marg="ufrechet", simu.cop="indep")
print(x)
summary(x)
plot(x, xlab="x", ylab="y")

#####
# (2) part of the workers' compensation dataset

x1 <- c(
  21.798086,  22.640528,  22.572010,  24.789710,  25.876764,  28.033613,
  22.525887,  12.004031,  12.713178,  13.596610,  14.811727,  12.774073,
  20.245789,  24.242468,  50.216515,  56.099793,  58.109747,  67.807105,
  73.852437,  84.208474,  83.604216,  19.507341,  20.810822,  23.838122,
  24.212193,  25.367578,  35.401344,  37.580989,  12.428727,  13.492474,
  23.471988,  24.101833,  24.766193,  26.078216)

x2 <- c(
 0.538707, 0.439184, 1.059775, 0.560013, 1.004997, 1.097314, 0.609833, 0.270222,
 0.229566, 0.596850, 0.196539, 0.134248, 0.489312, 0.418218, 0.769208, 0.649707,
 0.503919, 0.675466, 0.545745, 1.562266, 0.931762, 0.291125, 0.499927, 0.151084,
 0.141910, 0.300373, 0.119761, 0.141300, 0.377662, 0.169574, 0.243585, 0.061215,
 0.055272, 0.312816, 0.160196, 0.623029, 0.280707, 0.174422, 0.176666, 0.153907,
 0.605122, 0.664457, 0.348918, 0.370878)

obs <- dataRTDE(cbind(x1, x2))
obs
summary(obs)

plot(obs)
```

EPD *The Extended Pareto Distribution*

Description

Density function, distribution function, quantile function, random generation.

Usage

```
dEPD(x, eta, delta, rho, tau, log = FALSE)
pEPD(q, eta, delta, rho, tau, lower.tail=TRUE, log.p = FALSE)
qEPD(p, eta, delta, rho, tau, lower.tail=TRUE, log.p = FALSE,
    control=list())
rEPD(n, eta, delta, rho, tau)
```

Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If length(n) > 1, the length is taken to be the number required.
eta	first shape parameter.
delta	nuisance parameter.
rho, tau	second shape parameter.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, $P[X > x]$.
control	A list of control paremeters. See section Details.

Details

The extended Pareto distribution is defined by the following density

$$f(x) = \frac{1}{\eta} x^{-1/\eta-1}[1+\delta(1-x^{-\tau})]^{-1/\eta-1}[1+\delta(1-(1-\tau)x^{-\tau})]$$

for all $x > 1$ when parametrized by τ. However, a typical parametrization is obtained by setting $\tau = -\rho/\eta$, i.e.

$$f(x) = \frac{1}{\eta} x^{-1/\eta-1}[1+\delta(1-x^{\rho/\eta})]^{-1/\eta-1}[1+\delta(1-(1+\rho/\eta)x^{\rho/\eta})]$$

for all $x > 1$ when parametrized by ρ.

The control argument is a list that can supply any of the following components:

upperbound The upperbound used in the optimize function when computing numerical quantiles, default to 1e6.

tol the desired accuracy used in the optimize function when computing numerical quantiles, default to 1e-9.

Value

dEPD gives the density, pEPD gives the distribution function, qEPD gives the quantile function, and rEPD generates random deviates.

The length of the result is determined by n for rEPD, and is the maximum of the lengths of the numerical parameters for the other functions.

The numerical parameters other than n are recycled to the length of the result. Only the first elements of the logical parameters are used.

Author(s)

Christophe Dutang

References

J. Beirlant, E. Joossens, J. Segers (2009), *Second-order refined peaks-over-threshold modelling for heavy-tailed distributions*, Journal of Statistical Planning and Inference, Volume 139, Issue 8, Pages 2800-2815.

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Insurance: Mathematics and Economics

This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

Examples

```
#####
# (1) density function
x <- seq(0, 5, length=24)

cbind(x, dEPD(x, 1/2, 1/4, -1))

#####
# (2) distribution function

cbind(x, pEPD(x, 1/2, 1/4, -1, lower=FALSE))
```

FGM *The Eyraud Farlie Gumbel Morgenstern Distribution*

Description

Density function, distribution function, quantile function, random generation.

Usage

```
dFGM(u, v, alpha, log = FALSE)
pFGM(u, v, alpha, lower.tail=TRUE, log.p = FALSE)
qFGM(p, alpha, lower.tail=TRUE, log.p = FALSE)
rFGM(n, alpha)
```

Arguments

u, v	vector of quantiles.
p	vector of probabilities.

`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`alpha`	shape parameter.
`log, log.p`	logical; if TRUE, probabilities p are given as log(p).
`lower.tail`	logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise, $P[X > x]$.

Details

The FGM is defined by the following distribution function

$$C(u, v) = u * v * (1 + \alpha * (1 - u) * (1 - v))$$

for all u, v in [0,1] and α in [0,1]. When `lower.tail=FALSE`, `pFGM` returns the survival copula $P(U > u, V > v)$.

Value

`dFGM` gives the density, `pFGM` gives the distribution function, `qFGM` gives the quantile function, and `rFGM` generates random deviates.

The length of the result is determined by `n` for `rFGM`, and is the maximum of the lengths of the numerical parameters for the other functions.

The numerical parameters other than `n` are recycled to the length of the result. Only the first elements of the logical parameters are used.

Author(s)

Christophe Dutang

References

Nelsen, R. (2006), *An Introduction to Copula, Second Edition*, Springer.

Examples

```
#####
# (1) density function
u <- v <- seq(0, 1, length=25)

cbind(u, v, dFGM(u, v, 1/2))
cbind(u, v, outer(u, v, dFGM, alpha=1/2))


#####
# (2) distribution function

cbind(u, v, pFGM(u, v, 1/2))
cbind(u, v, outer(u, v, pFGM, alpha=1/2))
```

fitRTDE *Fitting a Tail Dependence model with a Robust Estimator*

Description

Fit a Tail Dependence model with a Robust Estimator.

Usage

```
fitRTDE(obs, nbpoint, alpha, omega, method="MDPDE", fix.arg=list(rho=-1),
    boundary.method="log", control=list())


## S3 method for class 'fitRTDE'
print(x, ...)
## S3 method for class 'fitRTDE'
summary(object, ...)
## S3 method for class 'fitRTDE'
plot(x, which=1:2, main, ...)
```

Arguments

`obs`	bivariate numeric dataset.
`nbpoint`	a numeric for the number of largest points to be selected.
`alpha`	a numeric for the power divergence parameter.
`omega`	a numeric for omega, see section Details.
`method`	a character string equals to `"MDPDE"`.
`fix.arg`	a named list of fixed arguments: either rho only e.g. `list(rho=-1)` or $rho, delta$ e.g. `list(rho=-1,delta=0)`.
`boundary.method`	a character string: either "log" or "simple", see section Details.
`control`	A list of control paremeters. See section Details.
`x, object`	an R object inheriting from `"fitRTDE"`.
`...`	arguments to be passed to subsequent methods.
`which`	an integer (1 or 2) to specify whether to plot eta or delta, respectively.
`main`	a main title for the plot.

Details

The function `fitRTDE` fits an extended Pareto distribution (η, τ are fitted while ρ is fixed) on the relative excess of Z_ω (see `zvalueRTDE`) using a robust estimator based on the minimum distance power divergence criterion (see `MDPD`). The boundary enforcement on η, τ is either done by the bounded BFGS algorithm (see `optim` with `method="L-BFGS-B"`) or by the bounded Nelder-Mead algorithm (see `constrOptim` with `method="Nelder-Mead"`) .

Value

`fitRTDE` returns an object of class `"fitRTDE"` having the following components:

`n` rownumber of `data`.

`n0` rownumber of `contamin`.

`alpha` a vector of `alpha` parameters.

`omega` a vector of `omega` parameters.

`m` a vector of `nbpoint`.

`rho` a numeric for `rho`.

`eta` estimate of eta.

`delta` estimate of $delta$.

`Ztilde` see `zvalueRTDE`.

Author(s)

Christophe Dutang

References

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Volume 57, Insurance: Mathematics and Economics

This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

Examples

```
#####
# (1) simulation

omega <- 1/2
m <- 48
n <- 100
obs <- cbind(rupareto(n), rupareto(n)) + rupareto(n)

#function of m
system.time(
x <- fitRTDE(obs, nbpoint=m:(n-m), 0, 1/2)
)
```

```
x
summary(x)
plot(x, which=1)
plot(x, which=2)
```

Frank *The Frank Distribution*

Description

Density function, distribution function, quantile function, random generation.

Usage

```
dfrank(u, v, alpha, log = FALSE)
pfrank(u, v, alpha, lower.tail=TRUE, log.p = FALSE)
qfrank(p, alpha, lower.tail=TRUE, log.p = FALSE)
rfrank(n, alpha)
```

Arguments

`u, v`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`alpha`	shape parameter.
`log, log.p`	logical; if TRUE, probabilities p are given as log(p).
`lower.tail`	logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, $P[X > x]$.

Details

The Frank is defined by the following distribution function

$$C(u,v) = -\frac{1}{\alpha}\log\left[1 - \frac{(1-e^{-\alpha u})(1-e^{-\alpha v})}{1-e^{-\alpha}}\right],$$

for all u, v in [0,1]. When `lower.tail=FALSE`, `pfrank` returns the survival copula $P(U > u, V > v)$.

Value

`dfrank` gives the density, `pfrank` gives the distribution function, `qfrank` gives the quantile function, and `rfrank` generates random deviates.

The length of the result is determined by `n` for `rfrank`, and is the maximum of the lengths of the numerical parameters for the other functions.

The numerical parameters other than `n` are recycled to the length of the result. Only the first elements of the logical parameters are used.

Author(s)

Christophe Dutang

References

Nelsen, R. (2006), *An Introduction to Copula, Second Edition*, Springer.

Examples

```
#####
# (1) density function
u <- v <- seq(0, 1, length=25)

cbind(u, v, dfrank(u, v, 1/2))
cbind(u, v, outer(u, v, dfrank, alpha=1/2))


#####
# (2) distribution function

cbind(u, v, pfrank(u, v, 1/2))
cbind(u, v, outer(u, v, pfrank, alpha=1/2))
```

Frechet *The Frechet Distribution*

Description

Density function, distribution function, quantile function, random generation.

Usage

```
dfrechet(x, shape, xmin, log = FALSE)
pfrechet(q, shape, xmin, lower.tail=TRUE, log.p = FALSE)
qfrechet(p, shape, xmin, lower.tail=TRUE, log.p = FALSE)
rfrechet(n, shape, xmin)

dufrechet(x, log = FALSE)
pufrechet(q, lower.tail=TRUE, log.p = FALSE)
qufrechet(p, lower.tail=TRUE, log.p = FALSE)
rufrechet(n)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`shape`	shape parameter.
`xmin`	lower bound parameter.
`log, log.p`	logical; if TRUE, probabilities p are given as log(p).
`lower.tail`	logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, $P[X > x]$.

Details

The Frechet distribution is defined by the following density

$$f(x) = shape * (x - xmin)^{(-shape-1)} * exp(-(x - xmin)^{(-shape)})$$

for all $x > xmin$. The unit Frechet distribution corresponds to `xmin=0` and `shape=1`.

Value

`dfrechet,dufrechet` give the density, `pfrechet,pufrechet` give the distribution function, `qfrechet,qufrechet` give the quantile function, and `rfrechet,rufrechet` generate random deviates.

The length of the result is determined by `n` for `rfrechet,rufrechet`, and is the maximum of the lengths of the numerical parameters for the other functions.

The numerical parameters other than `n` are recycled to the length of the result. Only the first elements of the logical parameters are used.

Author(s)

Christophe Dutang

References

Kotz, S. and Nadarajah, S. (2000), *Extreme Value Distributions: Theory and Applications*, Imperial College Press.

Beirlant, J., Goegebeur, Y., Teugels, J., Segers (2004), *Statistics of Extremes: Theory and Applications*, John Wiley and Sons.

Examples

```
#####
# (1) density function
x <- seq(0, 5, length=24)

cbind(x, dfrechet(x, 1/2, 1/4))
```

```
#####
# (2) distribution function

cbind(x, pfrechet(x, 1/2, 1/4))
```

MDPD *The Minimum Distance Power Divergence statistics*

Description

Computes the power divergence statistics then used a minimization problem.

Usage

```
MDPD(theta, densfun, obs, alpha, ..., control=list())
```

Arguments

theta	the parameter of the distribution given as a vector.
densfun	a function computing the theoretical density function.
obs	a numeric vector of observations
alpha	a numeric for the power divergence parameter.
...	further arguments to be passed to the density function.
control	A list of control paremeters. See section Details.

Details

The Power Divergence for a density function f and observations $X_1, ..., X_n$ is defined as

$$\Delta(f,\alpha) = \int_R f^{1+\alpha}(x)dx - \left(1+\frac{1}{\alpha}\right)\frac{1}{n}\sum_{i=1}^n f^\alpha(X_i)$$

for $\alpha > 0$

$$\Delta(f,0) = -\frac{1}{n}\sum_{i=1}^n \log f(X_i)$$

for $\alpha = 0$.

The `control` argument is a list that can supply any of the following components:

`eps` a small positive floating-point number used when `integrate` stalled, default to `1e-3`.

`tol` the desired accuracy used in the `integrate` function when computing the power divergence, default to `1e-3`.

`lower` the lower bound of the domain of the density function, default to 1.

`upper` the lower bound of the domain of the density function, default to infinity.

Value

MDPD returns the power divergence against the density function densfun as a numeric.

Author(s)

Christophe Dutang

References

Basu, A., Harris, I.R., Hjort, N.L., Jones, M.C., (1998). *Robust and efficient estimation by minimizing a density power divergence*, Biometrika, 85, 549-559.

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Insurance: Mathematics and Economics

This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

Examples

```
#####
# (1) small example

omega <- 1/2
m <- 10
n <- 100
obs <- cbind(rupareto(n), rupareto(n)) + rupareto(n)

#unit Pareto transform
z <- zvalueRTDE(obs, omega, nbpoint=m, output="relexcess")

MDPD(c(1/2, 1/4), dEPD, z$Z, alpha=0, rho=-1)
```

---

qqparetoplot *The QQ Pareto plot*

Description

Plot the quantile-quantile Pareto plot

Usage

```
qqparetoplot(x, ..., highlight=c("red","cross"))
```

Arguments

`x`	data vector.
`highlight`	character string used in `points` to plot outliers.
`...`	further arguments for `plot.default`.

Details

`qqparetoplot` plots the quantile-quantile Pareto plot and may highlight some points having name `"new"`.

Value

Invisible list with component `x` for the x-coordinates and `y` for the y-coordinates.

Author(s)

Christophe Dutang

Examples

```
#####
# (1) small examples

set.seed(1234)
x <- rupareto(100)
qqparetoplot(x)

x <- rexp(100)
qqparetoplot(x)
```

`RTDE` *Data object used for a Tail Dependence model*

Description

Data object used for a Tail Dependence model.

Usage

```
RTDE(obs=NULL, simu=list(), contamin=list(),
    nbpoint, alpha, omega, method="MDPDE", fix.arg=list(rho=-1),
    boundary.method="log", core=1, keepdata, control=list())


## S3 method for class 'RTDE'
```

```
print(x, ...)
## S3 method for class 'RTDE'
summary(object, ...)
## S3 method for class 'RTDE'
plot(x, which=1:3, FUN=mean, main, ...)

prob(object, q, ...)
## Default S3 method:
prob(object, q, ...)
## S3 method for class 'RTDE'
prob(object, q, ...)
```

Arguments

`obs`	bivariate numeric dataset.
`simu`	a names list with components: `"nb"`, `"marg"`, `"cop"`, `"replicate"`. When needed, `"cop.par"` must be provided, see `dataRTDE`.
`contamin`	a names list with components: `"eps"`, `"method"`, `"marg"`, `"cop"`. When needed, `"cop.par"` must be provided, see `dataRTDE`.
`nbpoint`	a numeric for the number of largest points to be selected.
`alpha`	a numeric for the power divergence parameter.
`omega`	a numeric for omega, see section Details.
`method`	a character string equals to `"MDPDE"`.
`fix.arg`	a named list of fixed arguments: either rho only e.g. `list(rho=-1)` or $rho, delta$ e.g. `list(rho=-1,delta=0)`.
`boundary.method`	a character string: either "log" or "simple", see section Details.
`core`	a numeric for the number of core to be used, only relevant for simulated data.
`keepdata`	a logical whether to return or not the dataset.
`control`	A list of control paremeters for `fitRTDE`.
`x, object`	an R object inheriting from `"RTDE"`.
`...`	arguments to be passed to subsequent methods.
`which`	an integer to specify what to plot: 1 eta, 2 delta, 3 probability estimates.
`FUN`	the function to be applied, default to `mean`.
`main`	a main title for the plot.
`q`	vector of quantiles.

Details

The function `RTDE` handles (empirical or simulated) data (cf. `dataRTDE`) and then fits a bivariate tail model using a `method` criterion (cf. `fitRTDE` and `MDPD`) based on an extended Pareto distribution approximation (`EPD`). Typical distributions for simulated data and/or contaminations are

- MarginalUnit Pareto upareto, Frechet Frechet.
- CopulaFrank Frank, FGM FGM.

For a good introduction, please refer to references.

Value

RTDE returns an object of class "RTDE" having the following components:

obs.type see dataRTDE.

data see dataRTDE.

fit see fitRTDE.

simu see dataRTDE.

contamin see dataRTDE.

setting a list summarizing the computation.

Author(s)

Christophe Dutang

References

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Volume 57, Insurance: Mathematics and Economics

This work was supported by a research grant (VKR023480) from VILLUM FONDEN and an international project for scientific cooperation (PICS-6416).

See Also

See fitRTDE for the fitting process and dataRTDE for the data-handling process.

Examples

```
#####
# (1) simulation

n <- 100
x <- RTDE(simu=list(nb=n, marg="ufrechet", cop="indep", replicate=1),
nbpoint=10:11, alpha=0, omega=1/2)
x
summary(x)
```

`upareto` *The unit Pareto Distribution*

Description

Density function, distribution function, quantile function, random generation.

Usage

```
dupareto(x, log = FALSE)
pupareto(q, lower.tail=TRUE, log.p = FALSE)
qupareto(p, lower.tail=TRUE, log.p = FALSE)
rupareto(n)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations. If `length(n) > 1`, the length is taken to be the number required.
`log, log.p`	logical; if TRUE, probabilities p are given as log(p).
`lower.tail`	logical; if TRUE (default), probabilities are $P[X \le x]$, otherwise, $P[X > x]$.

Details

The extended Pareto distribution is defined by the following density and distribution function

$$f(x) = \frac{1}{x^2}, F(x) = 1 - \frac{1}{x},$$

for all $x > 0$.

Value

`dupareto` gives the density, `pupareto` gives the distribution function, `qupareto` gives the quantile function, and `rupareto` generates random deviates.

The length of the result is determined by `n` for `rupareto`, and is the maximum of the lengths of the numerical parameters for the other functions.

The numerical parameters other than `n` are recycled to the length of the result. Only the first elements of the logical parameters are used.

Author(s)

Christophe Dutang

References

Johnson, N.L., Kotz, S. and Balakrishnan, N. (2000), *Continuous Univariate Distributions, Volume 1, Second Edition*, John Wiley and Sons.

Examples

```
#####
# (1) density function
x <- seq(0, 5, length=24)

cbind(x, dupareto(x))

#####
# (2) distribution function

cbind(x, pupareto(x))
```

---

zvalueRTDE *The Z-value random variable*

Description

Compute the Z-value variable from a bivariate dataset.

Usage

```
zvalueRTDE(obs, omega, nbpoint, output=c("orig", "relexcess"),
    marg=c("upareto", "ufrechet", "uunif"))

## S3 method for class 'zvalueRTDE'
print(x, ...)
## S3 method for class 'zvalueRTDE'
summary(object, ...)


relexcess(x, nbpoint, ...)
## Default S3 method:
relexcess(x, nbpoint, ...)
## S3 method for class 'zvalueRTDE'
relexcess(x, nbpoint, ...)
```

Arguments

`obs`	bivariate numeric dataset.
`omega`	a numeric for omega, see Details.
`nbpoint`	a numeric for the number of largest points to be selected.
`output`	a character string for the output: either `"orig"` for original value or `"relexcess"` for relative excess.
`marg`	a character string for the empirical margin transformation: either `"upareto"` for unit Pareto, `"ufrechet"` for unit Frechet or `"uunif"` for unit uniform margin.
`x, object`	an R object inheriting from `"zvalueRTDE"`.
`...`	arguments to be passed to subsequent methods.

Details

Given a bivariate dataset $(X_i, Y_i)_i$ of n points, two variables are defined: (1) for `output="orig"`, the $\widetilde{Z}_{\omega,i}$ variable

$$\tilde{Z}_{\omega,i} = \min\left(f\left(\frac{R_i^X}{n+1}\right), \frac{\omega}{1-\omega} f\left(\frac{R_i^Y}{n+1}\right)\right)$$

where $f(x)$ is the margin transformation and $i = 1, ..., n$; (2) for `output="relexcess"`, the Z_j variable

$$\frac{\widetilde{Z}_{\omega,n-m+j,n}}{\widetilde{Z}_{\omega,n-m,n}}$$

where m equals `nbpoint`, $j = 1, \ldots, m$, and $\widetilde{Z}_{\omega,1,n}, ..., \widetilde{Z}_{\omega,n,n}$ are the order statistics of $\widetilde{Z}_{\omega,1}, ..., \widetilde{Z}_{\omega,n}$. The margin transformation is

$$f(x) = \frac{1}{1-x}, f(x) = \frac{1}{-\log(x)}, f(x) = x,$$

respectively for unit Pareto (`marg="upareto"`), unit Frechet (`marg="ufrechet"`) and unit uniform margin (`marg="uunif"`).

Value

`zvalueRTDE` computes the Z-variable and returns an object of class `"zvalueRTDE"` having the following components `type` (either `"orig"` or `"relexcess"`), `omega`, `Ztilde` or `Z`, `n`, possibly `m`.

`relexcess` computes the relative excesses from a Z-variable and returns an object of class `"zvalueRTDE"` of type `"relexcess"`.

Author(s)

Christophe Dutang

References

C. Dutang, Y. Goegebeur, A. Guillou (2014), *Robust and bias-corrected estimation of the coefficient of tail dependence*, Volume 57, Insurance: Mathematics and Economics

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See Also

See fitRTDE for the fitting process and dataRTDE for the data-handling process.

Examples

```
#####
# (1) example

omega <- 1/2
m <- 10
n <- 100
obs <- cbind(rupareto(n), rupareto(n)) + rupareto(n)

#unit Pareto transform
zvalueRTDE(obs, omega, output="orig")

relexcess(zvalueRTDE(obs, omega, output="orig"), m)
zvalueRTDE(obs, omega, nbpoint=m, output="relexcess")
```

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